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ABSTRACT

In this paper, the problem of choosing between education with the usual teacher-student interaction and television-assisted education is considered. The method used to select the alternative with minimum costs is presented, discussed, and applied to Botswana. (Pages 5 and 10 may reproduce poorly.) (Author)

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DECISION MODEL FOR PLANNING TECHNOLOGICALLY ASSISTED EDUCATION: WITH APPLICATION TO BOTSWANA

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I. INTRODUCTION

The object of this paper is to apply and expand methods previously suggested in reference (1) for planning technological inovations in education. The problem to be considered in section II is that of minimizing the cost of providing instruction for the students that the educational system is expected to have between dates $j = 1$ and $j = J$. It will be assumed that two alternatives are being considered: a) normal teacher student interaction and b) teachers assisted by television. The methods to be presented can easily be adapted to planning more complex Television Assisted Education (TAE) systems.

In addition the basic ideas of the model to be presented can be used to plan other types of technologically assisted education. This possibility is explored in detail in reference (1), which is why it will not be presented below. In section III an extension of the model in section II is used to study whether TAE is economically feasible in Botswana.

II. THE BASIC MODEL

II-1. Flows of Students

The number of students that the educational system is expected to have between dates $j = 1$ and $j = J$ will be denoted with S_{ij} where i denotes grade, $i = 1,\ldots,I$ and j denotes date. Below it will be assumed that S_{i1}, $i = 1,\ldots,I$ and S_{1j}, $j = 1,\ldots,J$ are known, i.e., that the students in all grades at date 1 (S_{i1}) and entering students at all dates (S_{1j}) are known. In addition it will be assumed that:

$$(1) \qquad S_{i+1,j+1} = \sigma_i S_{ij} \qquad \begin{matrix} i = 1,\ldots,(I-1) \\ j = 1,\ldots,(J-1) \end{matrix}$$

where the transition coefficients σ_i are assumed to be known.

It should be observed that the knowledge of S_{11}, S_{1j} and σ_i in equation (1) is equivalent to that of S_{ij} for all i and j. Also it should be observed that other descriptions of the educational system could be used instead of (1). The advantage of this description is that it simplifies the presentation and can easily be replaced by more sophisticated descriptions such as those in references(1), and (2)

The next step is to specify the number of hours of education received by the students. It will be assumed here that h_s (datum) is the number of hours per student. As a consequence the number of periods of education received in year j - to be noted by L_j - is:

$$(2) \qquad L_j = \sum_{i=1}^{I} h_s S_{ij}$$

II-2. Teacher Requirements

The total number of periods received by the students must be related to the total offered by teachers alone or teachers assisted by television. For this let:

h_{tkij} denote the average number of periods (h_t) offered by each

k = 1 TAE teachers

k = 2 unassisted teachers

in grade i, year j

T_{kij} denote the number of teachers of type

k = 1 TAE teachers

k = 2 unassisted teachers

in grade i, year j.

The assumption introduced here that there is only one type of TV assisted teacher and one type of unassisted teacher can easily be removed at the cost of a more complicated notation. In section III a somewhat more complex case will be considered.

The average number of periods offered is related to the number of teachers in the following way:

$$h_{tkij} = \begin{cases} 0 \text{ if } T_{kij} = 0 \\ h^*_{tki} \text{ if } T_{kij} > 0 \end{cases}$$

This relationship formalizes the fact that if no teachers of type k are available, no periods will be offered by them. However any one teacher in grade i will offer h^*_{tki} periods, where h^*_{tki} is a known constant. The total number of hours offered by any one of these two types of teachers is $h^*_{tkij} T_{kij}$.

In order to transform the number of periods offered to periods received, according to reference (2) the number offered should be multiplied by the size of the classes, i.e.,

$$(3) \qquad L_j = \sum_{k=1}^{2} \sum_{i=1}^{I} c_{ki} h^*_{tki} T_{kij} \qquad j = 1,\ldots,J$$

where c_{ki} (data) denotes class size in grade i for teachers of type k.

In actual applications of TAE it is likely that some type of gradual introduction is desirable. This condition can be imposed on the model in several ways. Perhaps the simplest case can be described with the following equation:

$$(4) \qquad c_{1i+1} h^*_{t1i+1} T_{1i+1,j+1} \leq m c_{1i} h^*_{t1i} T_{1ij} \qquad i = 1,\ldots,I-1$$
$$j = 1,\ldots,J-1$$

where m is a large positive constant.

The constraint in (4) simply says that TAE cannot be introduced in any grade $i = 2,\ldots,I$ unless it is being used in the previous grade in the previous year. Similarly it would be possible to introduce television by subject so that all grades would receive a given subject before TAE is used for another subject. Both of these possibilities enable the education system to adjust gradually to television and allow time for curriculum development.

Another way to condition the model to a gradual introduction of TAE is to restrict the elimination of teachers of type 2. This can be accomplished with constraints of the form:

$$(5) \qquad T_{2ij} \geq \bar{B}_{2ij}$$

where $\bar{B}$'s are known constants. Frequently constraint (5) should be modified to express that the number of teachers of type 2 in all grades must be at least equal to a lower bound. The modification needed is quite obvious.

Similarly upper bounds, due to supply conditions, might constrain the number of either one or both types of teachers. These constraints might take the form:

$$(6) \qquad \hat{B}_{kij} \geq T_{kij}$$

Again this constraint can be modified if necessary.

It should be observed that not all the constraints (4) to (6) may be needed to insure an orderly transition.

II-3. Relationship Between Periods Offered and Television Transmission and Reception

In the type of system studied here it is assumed that assisted teachers attend a TV transmission together with their students. Once the transmission is finished the teacher remains in the class to answer the questions of the students, assign homework and go over assignments.

Let the average time each assisted teacher receives transmissions be h'_{tlij}. This variable satisfies the following conditions: $h'_{tlij} < h_{tlij}$, and $h'_{tlij} = h'^{*}_{t}$ whenever $h_{tlij} = h^{*}_{tli}$. The time h'_{tlij} is expanded by a factor x (datum) to the average time taught by assisted teachers, i.e., $h_{tlij} = x\, h'_{tlij}$.

The hours of television received per available set in grade i will be denoted with h_{vij}, and the number of sets available in grade i with V_{ij}. The following relationship between these two variables must be considered:

$$(7) \qquad h_{vij} = \begin{cases} 0 \text{ if } V_{ij} = 0 \\ h^{*}_{vi} \text{ if } V_{ij} > 0 \end{cases}$$

The total periods of reception are $h^{*}_{vi} V_{ij}$. This value is expanded to hours received by the students, i.e., to $c_{vi}\, h^{*}_{vi}\, V_{ij}$ where c_{vi}(datum) is the number of students per set in grade i. Finally

$$(8) \qquad c_{li}\, h^{*}_{tli}\, T_{lij} = x\, c_{vi} h^{*}_{vi}\, V_{ij}$$

The following relationship derived from (8) will be used

$$(9) \qquad V_{ij} = q_i\, T_{lij}$$

where the value of q_i should be obvious.

The left hand side of equation (8) is the number of periods of education received by the students defined in terms of the periods offered by the assisted teachers. The right hand side is the number of periods

received defined in terms of TV transmissions received in the different sets. It is clear that these two numbers must be equal as stated in equation(8).

Finally the number of hours that each one of the TV sets is used will be related to the hours of transmission. For this it will be assumed that each grade received a different transmission. To simplify it will be assumed also that all the transmissions to grade i are received in that grade with all the TV sets available to that grade. These assumptions imply that

$$(10) \qquad h_{vi} = N_{ij}$$

where N_{ij} is the number of hours of transmission to grade i in year j.

II-4. Costs of the System

A first impression is that unassisted education has only current costs, while TAE has both current and capital costs. However both human and physical capital have current and capital costs. This fact will be considered below.

Under certain assumptions it can be shown that the total discounted value of salaries paid to a teacher is equal to his value as human capital. If this is the case, salaries paid to a person cover both current and capital costs. Below, it will be assumed that this is case. The salaries paid to teacher of type k will be will denoted with w_k k=1,..., K.

It is also possible to express the current and capital costs of TV transmission and reception in one value e.g. annual payments. However current and capital costs should be considered explicitly since educational systems usually distinguish between the two. The cost of transmission including equipment, programming and actual transmission costs will be denoted by p_{1i}. The cost of reception will be denoted by p_{2i}.

It follows from the above that the total costs of assisted education in period j, grade i, will be:

(11) $$R_{1ij} = b_{ij} p_{1i} + v_{ij} p_{2i} + w_1 h_{t1ij} T_{1ij}$$

and that of unassisted education for the same grade and year is:

(12) $$R_{2ij} = w_2 h_{t2ij} T_{2ij}$$

II-5. The Minimization of Costs

With the formulae introduced above it is possible to state the problem of minimization of the cost of providing instruction to the students attending the educational system between dates 0 and J. At the risk of some repetition the problem will be stated in full.

The objective is to minimize:

(13) $$Z = \sum_{j=1}^{J} \sum_{i=1}^{I} \left[h_{vij} p_{1i} + (q_i p_{2i} + w_1 h^*_{t1i}) T_{1ij} + w_2 h^*_{t2i} T_{2ij} \right]$$

where formulae (9) and (10) have been used to simplify formulae (11) & (12).

This minimization should be made subject to the following constraints:

(14) $$L_j = \sum_{k=1}^{2} \sum_{i=1}^{I} c_{ki} h^*_{tki} T_{kij} \qquad j = 1,\ldots,J$$

(15) $$c_{1i+1} h^*_{t1i+1} T_{1i+1,j+1} \leq m\, c_{1i} h^*_{t1i} T_{1ij} \qquad i = 1,\ldots,I-1 \quad j = 1,\ldots,J-1$$

(16) $$T_{2ij} \geq \bar{B}_{2ij} \qquad i = 1,\ldots,I \quad j = 1,\ldots,J$$

(17) $$\hat{B}_{kij} \geq T_{kij} \qquad i = 1,\ldots,I \quad j = 1,\ldots,J$$

(18) $$h_{vij} = \begin{cases} 0 \text{ if } T_{1ij} = 0 \\ h^*_{vi} \text{ if } T_{1ij} > 0 \end{cases}$$

The problem as stated cannot be solved with the available techniques of mathematical programming. The characteristics of constraints (18) are responsible for this. In order to make it more treatable this constraint will be transformed in the following way. Let:

$$(19) \qquad d_{ij} = \begin{cases} 0 \text{ if } T_{1ij} = 0 \\ 1 \text{ if } T_{1ij} > 0 \end{cases}$$

Then the expression $h_{vij} P_{1i} d_{ij}$ should replace $h_{vij} P_{1i}$ in the objective function. The following two constraints should then be added:

$$(20) \qquad T_{1ij} - r_{1ij} d_{ij} \leq 0 \qquad \begin{matrix} i = 1,\ldots,I \\ j = 1,\ldots,J \end{matrix}$$

where r_{1ij} an arbitrary upper bound of T_{1ij}, (the subscript 1 in r denotes the first computational round), and

$$(21) \qquad 0 \leq d_{ij} \leq 1 \qquad \text{and} \qquad d_{ij} \text{ are integers.}$$

The main advantage of the trarsformation described is that it reduces the problem in (13) to (16) to one of mixed integer-continuous variable programming. There are standard techniques to solve this type of problem, however, they do not seem to be particularly efficient (3). Therefore the following computational alternative will be suggested.

First solve the problem in (13) with the above changes, (19) to (21), ignoring the condition that d_{ij} should be integers. Assume that in this solution there are some d_{ij}, to be denoted with d^{*}_{ij}, that are different from zero, but less than one. Then for these d^{*}_{ij} define

$$r_{2ij} = r_{1ij}\, d^*_{ij}$$

and solve the linear programming problem again using r_{2ij} in place of r_{1ij} still ignoring the condition that d_{ij} should be integers. These iterations should be repeated as long as some d_{ij} are not zero or one.

II APPLICATION TO BOTSWANA

Below the model in section I will be used to study whether it is economical and advisable to use TAE in the secondary schools of Botswana, during the period from 1976 to 1980, i.e., J = 5.

Secondary schools in Botswana have 5 grades called forms, i.e., I = 5. The forecasted number of students in each grade appear in table I. To compute the number of periods per student the fact that classes meet for 36 weeks per year, with 3 additional weeks for exams, and that each student takes 40 classes of 40 minutes each per week will be used with this:

$$h_s = 40 \times 36 = 1440$$

The estimated number of periods of education received appears in table II.

Two types of teachers are currently used in the secondary schools of Botswana: nationals with secondary school and teacher training, and college graduates who are mostly expatriates. In order to improve the quality of education it has been proposed to assist the trained nationals with television, or to replace them with college graduates. This means that the analysis below deals with one type of TAE teachers and two types of unassisted teachers: those with college degrees and teacher trained nationals. The numbers of those teachers will be denoted with T_{kij}, k = 1,...,3, where the subscript k stands for the type of teacher.

It will be assumed below that assisted teachers will teach 32 periods per week during 36 weeks ($h_{c1i} = 32 \times 36 = 1152$), while unassisted teachers teach an average of 26.66 periods per week ($h_{tki} = 26.66 \times 36 = 960$ k = 2,3). It will be assumed that the average class size for forms 1 to 5 is 35, ($C_{ki} = 35$ k = 1,3 , i = 1,5).

The supply of teachers with high school education is restricted to those already existing minus retirements and deaths plus the new graduates of teacher training colleges. The expected supply appears in table III. In this case the constraint takes the form:

(22) $\sum_{i=1}^{5} (T_{1ij} + T_{3ij}) \leq$

no constraint will be assumed with respect to the number of qualified graduates that could offer their services as teachers. The reason for this is that as observed they are mostly expatriates and any number of them could be hired if sufficient financial resources are available.

An additional restriction will be imposed on the number of unassisted trained nationals. The reason for this is to assure that they are eliminated by 1980. This constraint takes the form:

$$(23) \quad \sum_{i=1}^{5} T_{3ij} \leq V_j$$

where V_j is a policy variable with $V_5 = 0$. Its values appear in table III.

Several arrangements can be assumed with respect to the location of the television sets. Below it will be assumed that not all of the classes have sets, and that each set is used for 32 periods per week. However, any one class will receive only 1/3 of the class periods from the television with the rest of the time being spent in direct contact with the classroom teachers. To make possible this arrangement, after a class and their teacher have received a transmission they will leave the classroom equiped with television to make way for another class. For this a sufficiently large group of students is required per school, as well as the rebroadcasting of some programs. With the assumptions above it follows that:

$$h_{vij} = 36 \times 32 = 1152$$

In addition it will be assumed that each equipped room will have only one set. As a consequence of this $c_{1i} = c_{vi}$, and equation (8) takes the form:

$$c_{1i} \times 36 \times 32 \times T_{1ij} = 3 \times c_{vi} \times 36 \times 32 \times V_{ij} \qquad \begin{matrix} i = 1,\ldots,5 \\ j = 1,\ldots,5 \end{matrix}$$

i.e. $q_{ij} = .33$ for all i & j.

The required data on costs appear in table IV. With respect to the yearly salaries for expatriate teachers it should be observed that they are mainly volunteers and contract teachers who receive a subsidy from England.

To complete the presentation of the model as applied to Botswana, it should be observed that the objective function in equation (13) should be altered to include the costs of three types of teachers. The model appears in table V..

The results of three succesive computations appear in Table VI. They show that despite that teachers of type three are apparently less expensive, they should be assisted as soon as posible with TV, or substituted with graduate teachers.

To interprete this results it should be observed that according to constraint (23), teachers of type 3 have to be substituted during the period of the plan by either Tv assited teachers or expatriates with colege education.

The first iteration shows that without paying all TAE costs , assisted teachers are less expensive that unassisted ones. The reason for thi s is that they teach more periods per week.

The second iteration gives a some what surprising result. TV assisted teachers are more expensive years 76 and 77 but less expnesive years 78 to 80. This is surprinsing because costs do not change during the period. The reason for the result is the following. TV asisted teacher are less expensive than those with college degree if a large number of them can be used. This is explained by the fact that Tv transmisions are a large fixed dost. As a consequence only large numbers of assisted teachers bring down the average cost of them. In years 76 and 77 the number of teachers that can be assisted with TV is not large enough to make it less expensive to use them. As a consequence, as many teachers with only high school education and without assistance as posible shoud be used. In the final years of the plan, the supply constrait for teachers with only high school education is less restrictive. As a consequence a large enough number of them can be used to justify the fixed TV costs.

The third iteration shows that the posibility of not using any TV assisted teachers in years 76 and 77 is more expensive than the one of giving TV assistance to all the the teaches without college education.

TABLE I

Number of Students by Grade and Year for Botswana

year \ grade i	j	1	2	3	4	5	Total
1976	1	2800	2590	2170	945	700	9205
1977	2	2870	2730	2310	1050	875	9835
1978	3	2940	2800	2450	1120	980	10290
1979	4	3010	2870	2520	1190	1050	10640
1980	5	3080	2940	2590	1260	1120	10990

Source: elaborated from data in The National Development Plan 1970-75.

TABLE II

Year	j	no. of students	h_s	L_j
1976	1	9205	1440	13 225 200
1977	2	9835	1440	14 162 400
1978	3	10290	1440	14 817 600
1979	4	10640	1440	15 322 600
1980	5	10990	1440	15 827 600

Source: abstracted from table II and text

TABLE III

Upper Limits on the Number of Unassisted Trained Nationals and Upper Limits on Total Supply of Trained Naturals

Year	j	$\hat{B}_j$	V_j
1976	1	120	96
1977	2	150	90
1978	3	210	84
1979	4	270	54
1980	5	330	0

Source: V_j - Arbitrarily set as a policy of the proposal

$\hat{B}_j$ - Based on suggestions in the National Plan

TABLE IV

Costs (in dollars)

Yearly Teachers Salaries	Type k	$w_k \times h_{tki}$
Television assisted nationals	1	1200
College graduates	2	4000
unassisted nationals	3	1200

Annual Transmission and Reception Costs

Type	h	$h^{*}_{vij} p_h$ ©
Broadcasting		
capital, recurrent	1	150 000
Reception	2	225

© values assumed valid for all i and j.

Sources: Teachers salaries: information provided by American volunteers recently returned from Botswana.

Television Costs: Estimated from the National Audio-visual Catalogue 1972

15

TABLE V

Variables:	T_{11}	T_{21}	T_{31}	T_{12}	T_{22}	T_{32}	T_{13}	T_{23}	T_{33}	T_{14}	T_{24}	T_{34}	T_{15}	T_{25}	T_{35}
Objective Function	1275	4000	1200	1275	4000	1200	1275	4000	1200	1275	4000	1200	1275	4000	1200
Eq (3) j=1	40320	33591	33591												
j=2				40320	33591	33591									
j=3							40320	33591	33591						
j=4										40320	33591	33591			
j=5													40320	33591	33591
Eq (23) j=5															1
j=4												1			
j=3									1						
j=2						1									
j=1			1												
Eq (22) j=1	1		1												
j=2				1		1									
j=3							1		1						
j=4										1		1			
j=5													1		1
Eq (20) j=1	1														
j=2				1											
j=3							1								
j=4										1					
j=5													1		
Eq (21) j=1															
j=2															
j=3															
j=4															
j=5															

TABLE V

T_{13}	T_{14}	T_{24}	T_{34}	T_{15}	T_{25}	T_{35}	d_1	d_2	d_3	d_4	d_5	
4000	1275	4000	1200	1275	4000	1200	150000	150000	150000	150000	150000	
												= 13 225 200
												= 14 162 400
33591												= 14 817 600
	40320	33591	33591									= 15 322 600
				40320	33591	33591						= 15 827 600
						1						= 0
			1									≤ 54
1												≤ 84
												≤ 90
												≤ 96
												≤ 120
												≤ 150
1												≤ 210
	1		1									≤ 270
				1		1						≤ 330
							-394					≤ 0
								-421				≤ 0
									-441			≤ 0
	1									-456		≤ 0
				1							-471	≤ 0
							1					≤ 1
								1				≤ 1
									1			≤ 1
										1		≤ 1
											1	≤ 1

Table VI

Number of Teachers of different types that should be used to minimize each.

		T_1	T_2	T_3	d_j
First Iteration (r_1)					
	1976	120	249	0	.304
	1977	150	241	0	.356
	1978	210	189	0	.476
	1979	270	132	0	.592
	1980	330	75	0	.700
Second Iteration (r_2)					
	1976	24	268	96	.200
	1977	60	259	96	.402
	1978	210	189	0	1.000
	1979	270	132	0	1.000
	1980	330	76	0	1.000
Third Iteration (r_3)					
	1976	120	274	0	1.000
	1977	150	271	0	1.000
	1978	210	189	0	1.000
	1979	270	132	0	1.000
	1980	330	76	0	1.000

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